**Supplementary file 2 - Description of the conditional moment test**

By noting that the first process of the hurdle model involves only the random variable 𝑀, while the second process uses the variable , the conditional moment test is constructed on the following statements. If the separation hypothesis is respected, the following moment conditions hold for the first and second processes:

The first equality cannot hold for the zero-inflated distribution since . The second equality holds for both zero-inflated and hurdle models because is identically 1, since . However, = is only correct for a hurdle distribution and cannot hold for a zero-inflated distribution. In this situation, the parameters of the first and second processes of the hurdle distribution can be consistently estimated with the condition , by the generalized method of moment (GMM):

(3)

(4)

In our numerical example, is a (23 × 1) vector and a (23 × 1) vector. Therefore, having evaluated the parameters of the hurdle distribution by the last two equations using a generalized method of moments, the test can be done by checking the following equality :

Using , under the null hypothesis that the model is correctly specified and that the separation hypothesis holds, explicit equations of this test are, following the notations of Prieger (2003):

with:

The conditional moment test is asymptotically and the condition is rejected at significance level 𝛿 when , with 𝑟 the number of restrictions tested in the model. Rejection of the test indicates model misspecification, where the parameters of the distributions are not correctly estimated by equations (3) and (4).